Numerical simulation of local instabilities of membranes reinforced with cables

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ABSTRACT: Numerical algorithm to model large deformations of cable reinforced membrane systems is considered in this paper. A local instability may develop in a membrane lacking compressive, bending and twisting rigidity due to the creation and propagation of uni- or bi- directional wrinkles. In addition, slip of reinforcing cables in such systems may trigger certain additional undesired local effects. These instabilities are preliminarily analyzed and modeled in this work. In the case of wrinkling a concept of cable analogy (Stanuszek 2003) is applied. Combination of this technique with the penalty function method (Aliabadi & Brebbia 1993) is used to model contact effects. In particular a deformation of flat rectangular membrane reinforced with edge cable tucked into specially prepared edge sleeve is simulated in the paper. The numerical model was based on Finite Element discretization. Certain qualitative comparisons with experimental results confirmed the efficiency of proposed technique.

1 INTRODUCTION

Cable reinforced membrane systems are often used as the covering systems of very large areas. Such systems are often made of a composite fabric and rubber materials, or translucent plastic sheets reinforced with steel or fibre cables. These cables may be mated with a membrane as members of the composite material (conglomerate) or as separate members of the system, bound with the membrane through friction forces. Such elastic structures carry only tension and work in a plane or uniaxial stress state. They cannot withstand compression or bending and if that happens, wrinkling zones and cable slack appear. This phenomenon occurs in local areas and has a great influence on the final deformed shape of the system (Stanuszek 2002).

Traditionally in case of fabric roof design the main emphasis has been laid on the fabric or the membrane component, with a little attention being given to the cable components present in the majority of tensile structures. Edge cables are commonly used to gather the tensile forces from a membrane and redirect these distributed surface forces to conveniently located and isolated anchorage points at its top or its foundation levels. In such case the collaboration of a membrane and cables should be considered and a possibility of cable slip investigated.

2 PROBLEM FORMULATION

Let us focus attention on the system presented on Figure 1. A thin rectangular membrane is made of rubber, and is reinforced by fibre cable guided along its edge in a special sleeve. The membrane and cable materials differ in the tensional rigidity. This difference is the main cause of the possible slip. Moreover, it is obvious to note that slipping of cable is possible only if the membrane starts to wrinkle.

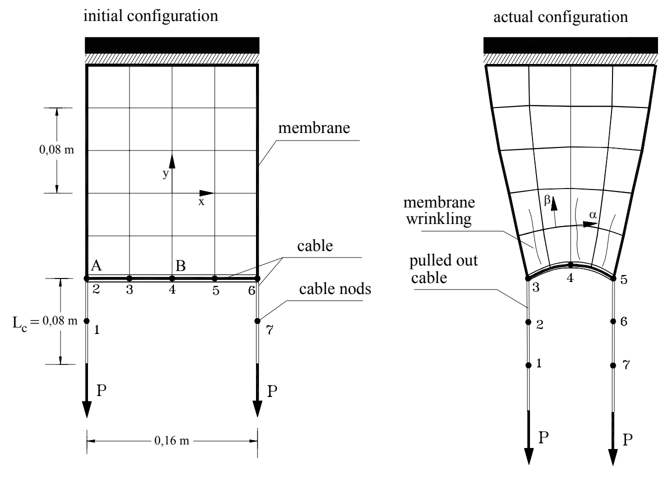
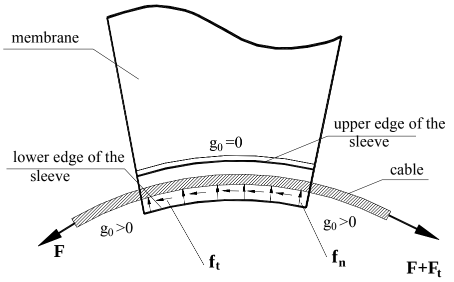


Figure1. Membrane reinforced cables - cable slip with membrane wrinkling.

This example shows the coupling of these two phenomena: slip and wrinkling. The second phenomenon was already modeled by Stanuszek (2003) with a new concept of so called *cable analogy*. This technique has been implemented here in the incremental algorithm, which additionally allows for modeling of the cables slip.

Geometrical nonlinearity of the problem requires taking into consideration the high terms of the Cauchy geometric equations. This results in need to distinguish between the initial and actual configurations of the system (Fung 1965). In the proposed model the contact force F (Fig.2.) between cable and membrane reaches the critical value and thus breaks the adhesion constraints. Then cable slip follows the stress redistribution in cable and membrane. It is worth to point out that in our considerations the range of contact zone is known.

Mathematical description of the problem with unilateral constrains is based on the variational inequality (Kikuchi & Oden 1988). The condition of normal contact of bodies $g_n = 0$ (g_n- a distance of bodies) restricts relative movements. As a result of interaction, the contact pressure f_n appears and the contact criteria take the form of the Kuhn-Tucker conditions.



$$g_n \geq 0 \text{ and } g_n \cdot f_n = \begin{cases} g_n > 0, f_n = 0; \text{no contact} \\ g_n = 0, f_n < 0; \text{ideal contact} \end{cases}$$

$g_n < 0$ body penetration

Figure 2. Distribution of internal forces within membrane sleeve with cable inside.

Total potential energy of the bodies in contact includes additional terms resulting from a normal (g_n) and a tangent (g_t) contacts. This energy is formulated under the assumption of admitted solution in the form:

$$\Pi(\mathbf{u}^1,\mathbf{u}^2,f_n) = \sum_{k=1}^{2}\left[\int_{\Omega^k} W^k \mathrm{d}\Omega_k - \int_{\Omega^k}\mathbf{b}^{\mathrm{T}k}\cdot\mathbf{u}^k\,\mathrm{d}\Omega^k - \int_{\partial\Omega_t^k}\mathbf{t}^{\mathrm{T}k}\cdot\mathbf{u}^k\,\mathrm{d}\Omega_t^k\right] - L_c \quad (1)$$

where $\mathbf{b}^k=\mathbf{b}^k(\mathbf{x}),\mathbf{x}\in\Omega^k$; k=1,2 and $\mathrm{t}^k=\mathrm{t}^k(\mathbf{x}_t),\mathbf{x}_t\in\Omega_t^k$ are the mass and the external distributed forces acting on bodies Ω^1, Ω^2 remaining in contact respectively. In (1) the vector $\mathbf{u}^k=\mathbf{u}^k(\mathbf{x})$ depicts deformation of the bodies and W^k represents density of the internal energy of the bodies. However the external work of the contact forces L_c includes the energy components resulting from the normal L_{gn} and tangent L_{gt} contact. Thus slip of a body (cable) is possible on distance g_t in the tangent direction and components of virtual work may be presented in the form:

$$L_c = L_{g_n} + L_{g_t} \quad (2)$$

where

$$L_{g_n} = \int_{\partial\Omega_c} f_n g_n(\mathbf{u}^1,\mathbf{u}^2)\,\mathrm{d}\Omega_c\ ;\ L_{g_t} = \int_{\partial\Omega_c} f_t g_t(\mathbf{u}^1,\mathbf{u}^2)\,\mathrm{d}\Omega_c \quad (3)$$

To simplify our considerations the Coulomb friction law was applied to calculate the correct tangent stresses in the form $|f_t| = -\mu|f_n|$, where μ is a friction coefficient.

3 MODEL OF CABLE REINFORCED MEMBRANE WITH SLIP ALLOWED

In the case of mating of a membrane and a cable in the system as presented in Fig.1 the penetration case is excluded (the cable either remains within the membrane sleeve or gets out of it). The required contact condition takes the form:

$$g_n \geq 0 \text{ and } g_n = u_n^2 - u_n^1 + g_0 \geq 0 \quad (4)$$

where $u^1{}_n$, $u^2{}_n$ denote displacements normal to the surface in the area of contact, for bodies 1 and 2 respectively; g_0 is the initial distance between points within the predicted contact zone of analyzed bodies. In case of $g_0 = 0$ a join and a slip modes are described by the following equations:

$$g_n = 0,\ u_n^2 - u_n^1 = g_0 \quad \text{and} \quad u_t^2 - u_t^1 = 0 \quad (5)$$

$$\text{or } \mathbf{f}_t = \mathrm{sgn}(\dot{u}_t^2 - \dot{u}_t^1)\,\mu_C\,\mathbf{f}_n \text{ where } \mathbf{F} = \int_{\Omega_c}\mathbf{f}_t\, d\Omega_c$$

There are generally two techniques used to deal with the unilateral constrains. These are the technique of Lagrange multipliers (Belytschko & Neal 1991, Nour-omid & Wriggers 1986) and the penalty function method (Aliabadi & Brebbia 1993). The second one was applied in our analysis and the contact forces were assumed to be proportional to the penetration distance with the penalty function c_n. For the coupled nodes in the contact zone the virtual work of contact forces in element $(.)_e$ can be expressed as:

$$L_{ce} = \frac{c_n}{2}(u_n^2 - u_n^1 + g_0)^2 \text{ with variation} \quad (6)$$

$$\delta L_{ce} = c_n(u_n^2 - u_n^1 + g_0)\delta u_n^2 - c_n(u_n^2 - u_n^1 + g_0)\delta u_n^1$$

After further calculations one can obtain the vector of contact forces in the form:

$$\mathbf{f}_{ne} = -\mathbf{k}_{ce}\begin{Bmatrix}\mathbf{u}^1\\ \mathbf{u}^2\end{Bmatrix} + \mathbf{g}_e \quad (7)$$

$$\textit{where}\quad \mathbf{k}_{ce} = \begin{bmatrix}\mathbf{k} & -\mathbf{k}\\ -\mathbf{k} & \mathbf{k}\end{bmatrix} \quad \textit{and} \quad \mathbf{g}_e = \begin{Bmatrix}-\mathbf{r}\\ \mathbf{r}\end{Bmatrix}$$

$\mathbf{k}_{ce}$ is the stiffness matrix of a contact element made of membrane and cable nodes coupled in the initial configuration. After aggregation of all internal and external loads over all system elements the total incremental equilibrium equation can be written as:

$$\delta\mathbf{U}_{\mathrm{I}} - \delta\mathbf{J}_{\mathrm{I}} = (\mathbf{K}_E + \mathbf{K}_G + \mathbf{K}_c)\delta\mathbf{u}_{\mathrm{I}} \qquad (8)$$

Strong nonlinearity of the problem requires the use of the Newton-Raphson algorithm with a new *trace path technique* (Stanuszek 2003). The complete form of presented matrices and techniques may be finding out in monograph (Stanuszek 2005).

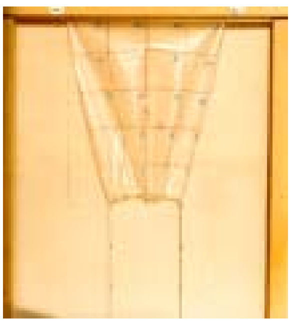
4 EXPERIMENTAL AND NUMERICAL TESTS

The algorithm presented above was implemented numerically within NAFDEM (Nonlinear Finite Difference and Element Methods) system (Stanuszek 2002), using Finite Element methodology. This is a very sophisticated numerical tool for discrete analysis of boundary value problems in local and global formulations.
To provide a better understanding of considered deformation mechanism (Fig.1) the experimental study of deformation of rectangular membrane was undertaken first. The physical model of the system was presented in Figure 3a.

(a) without loads.

(b) 2P = 1N.

(c) 2P = 2N.



(d) 2P = 4N.

Figure 3. Experimental study of wrinkling of plastic membrane with cable slip allowed.

The deformation process and obtained results were presented in photos (Figures 3b-d) and in table 1.

Table1. Experimental results – cable slip (Fig.1).

No.	Force 2P [N]	Displacement x of node A [m]	Displacement y of node A [m]	Displacement y of node B [m]	Cable length L_c [m]
1	0	0	0	0	0,080
2	1	0,003	-0,002	0,0005	0,088
3	2	0,019	-0,001	0,003	0,101
4	4	0,042	0,001	0,010	0,120
5	5	0,044	0,003	0,013	0,125

Next the lifting process of the rectangular membrane was analyzed. The initial configuration of the membrane is presented in Fig. 4. It is worth to point out that the initial configuration of the presented system is flat and singular in the case of loading in the normal direction. Such a case requires the application of so called *artificial pre-stress*, which influences only the iteration path (Fig. 5a,b) while reaching the equilibrium.

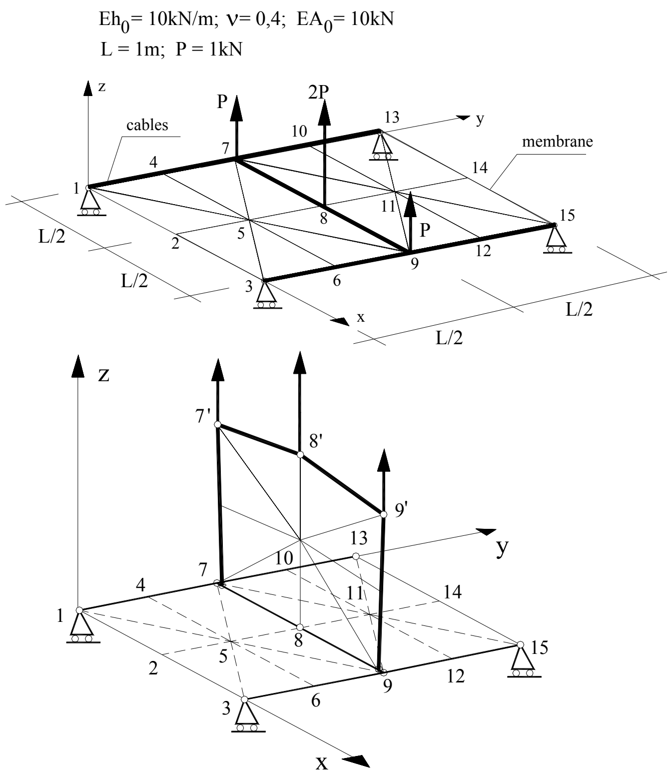


Figure 4. A cable reinforced membrane – lifting of the membrane.

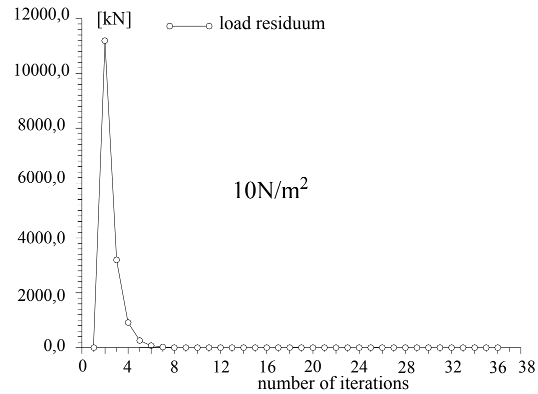


Figure 5a. The iteration process for pre-stressing of 10N/m^2.

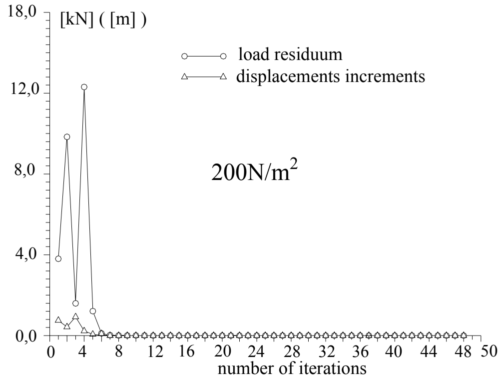
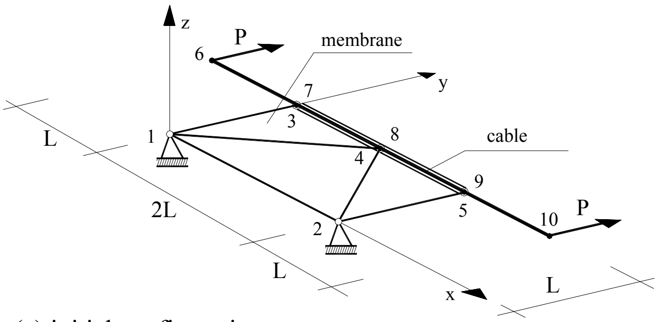
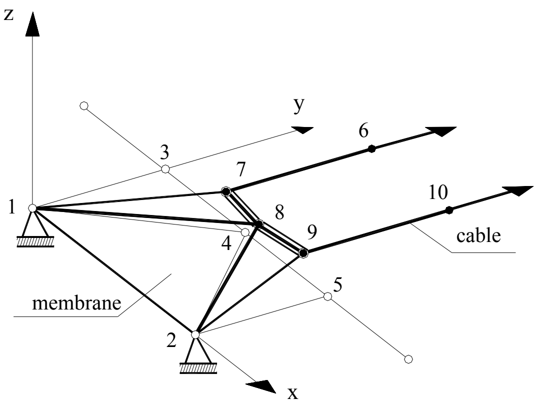


Figure 5b. The iteration process for pre-stressing of $200 N/m^2$.

Slip of a single cable in a cable reinforced structure was considered next. The initial configuration of the system is presented in Figure 6a.



(a) initial configuration.



(b) deformed configuration.

Figure 6. A cable reinforced membrane – deformation with friction effects included.

The structure was analyzed as frictionless as well as with frictional contact assumptions. In the first case the nodes 7 and 9 converge to the node 8 whereas in the second one the final configuration due to equilibrium of the system with additional contact forces taken into consideration is shown in Fig. 6b.
A strong nonlinearity of the problem required the implementation of an iteration algorithm using new *trace path technique*. The iterative process in case of assumed friction was presented in details in Figure 7. Obtained results were qualitatively verified with previously presented experimental study and engineering intuition. More precise comparison was impossible due to the unavailability of physical constants for experimentally examined membrane and cable (Fig. 1).

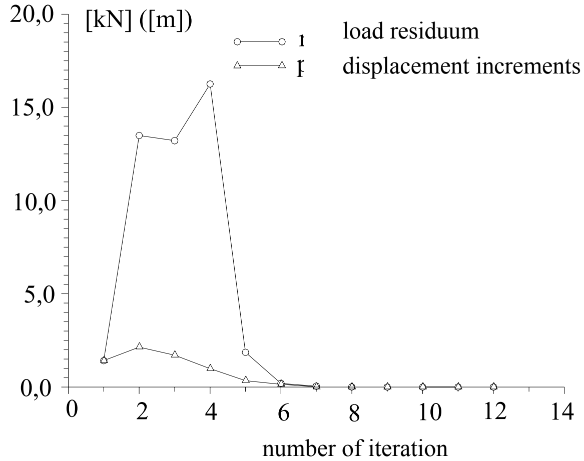


Figure 7. The course of iteration in case 6b.

5 CONCLUSION

The preliminary analysis of the frictional contact-slip behavior between a wrinkled membrane and cables in reinforced membrane system was presented. Special attention was paid to the numerical simulation of a very complex deformation of lifting up of cable reinforced membrane with singular initial configuration. In such a case the technique of *artificial pre-stress* turned out very effective. Next, modeling of the deformation of simple cable reinforced rectangular membrane with cable slip allowed was undertaken. The numerical results obtained agreed qualitatively with experimental ones and engineering intuition.

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